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The objective of this project was to derive global convergence theorems and practical numerical algorithms for probability-one homotopies in the area of constrained optimization, large sparse unstructured systems of nonlinear equations, H_2 control design and analysis, and H_2/H_infinity disturbance and rejection controls. The emphasis was on useful convergence theorems expressed in terms of conditions on the original problem rather than conditions on the derived problem. Important applications were considered as a guide to what theoretical questions should be asked.

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FINAL TECHNICAL REPORT FOR AFOSR GRANT F49620-92-J-0236
THEORY AND APPLICATION OF HOMOTOPY TECHNIQUES IN NONLINEAR PROGRAMMING AND CONTROL SYSTEMS

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The objective of this project is to derive global convergence theorems and practical numerical algorithms for probability-one homotopies in the areas of constrained optimization, large sparse unstructured systems of nonlinear equations, H_2 control design and analysis, and H_2/H_∞ disturbance rejection controls. The emphasis is on useful convergence theorems expressed in terms of conditions on the original problem rather than conditions on the derived problem. Important applications are considered as a guide to what theoretical questions should be asked.

The combined H^2/H^∞ model reduction problem.

The problem of finding a reduced order model, optimal in the H^2 sense, to a given system model is a fundamental one in control system analysis and design. The addition of a H^∞ constraint to the H^2 optimal model reduction problem results in a more practical yet computationally more difficult problem. Several order reduction techniques had been proposed for approximating the frequency response of a given system. For example, frequency weighting had been studied by D. F. Enns in conjunction with balancing (B. C. Moore). Moreover, Hankel norm reduction had been shown to have fundamental ramifications for frequency domain approximation. There were theoretical formulations for the combined H^2/H^∞ model reduction problem, but there had been no serious numerical studies.

The combined H^2/H^∞ model reduction problem is formulated as: given the controllable and observable, time invariant, continuous time system

$$\dot{x}(t) = A\,x(t) + B\,Du(t), \qquad y(t) = C\,x(t),$$

where $t \in [0,\infty)$, $A \in E^{n\times n}$ is asymptotically stable, $B \in E^{n\times m}$, $C \in E^{l\times n}$, $D \in E^{m\times p}$ $(m \le p)$ and the input $Du(t)$ is white noise with symmetric and positive definite intensity $V \equiv DD^T$, find a n_m-th order model $(n_m < n)$

$$\dot{x}_m(t) = A_m\,x_m(t) + B_m\,Du(t), \qquad y_m(t) = C_m\,x_m(t),$$

where $A_m \in E^{n_m\times n_m}$, $B_m \in E^{n_m\times m}$, $C_m \in E^{l\times n_m}$, which satisfies the following criteria:

(i) A_m is asymptotically stable;

(ii) the transfer function of the reduced order model lies within γ of the transfer function of the full order model in the H_∞ norm, i.e., $\|H(s) - H_m(s)\|_\infty \le \gamma$ where $H(s) \equiv EC(sI_n - A)^{-1}BD$, $H_m(s) \equiv EC_m(sI_m - A_m)^{-1}B_mD$, $\gamma > 0$ is a given constant, $E \in E^{q\times l}$ $(q \ge l)$ is a given constant matrix; and

(iii) the H^2 model reduction criterion $J(A_m, B_m, C_m) \equiv \lim_{t\to\infty} \mathcal{E}\left[(y - y_m)^T R(y - y_m)\right]$ is minimized, where $\mathcal{E}$ is the expected value and $R = E^T E$ is a symmetric and positive definite weighting matrix.

Several approaches based on homotopy methods had been proposed. The issues were the number of degrees of freedom, the well posedness of the finite dimensional optimization problem, and the numerical robustness of the resulting homotopy algorithm. Homotopy algorithms based on several formulations — input normal form; Ly, Bryson, and Cannon's 2×2 block parametrization; a new nonminimal parametrization — were developed and compared. This represented an attempt to design practical and efficient numerical algorithms for solving the combined H^2/H^∞ model order reduction problem.

One of the main conclusions for this study was that the more degrees of freedom that a formulation used, the more robust was the resulting numerical algorithm. The input normal form and Ly form homotopies were very efficient for the combined H^2/H^∞ model reduction problems. However, they might fail to exist or be very ill conditioned. The over-parametrization formulation solved the ill conditioning issue, but introduced singularity at the solution and might fail for a high dimensional system, which would inevitably have a high order singularity at the solution.

Another conclusion was that solving the H^2 optimal model order reduction problem might be well worth the effort (compared to simple balancing). The studies also proved the worth of adding the H^∞ constraint. Finally, globally convergent homotopy methods were a viable approach to the computationally very difficult combined H^2/H^∞ model order reduction problem.

LQG control synthesis with an H^∞ performance bound.

The H^2/H^∞ mixed-norm controller synthesis problem provides the means for simultaneously addressing H^2 and H^∞ performance objectives. In practice such controllers provide both nominal performance (via H^2) and robust stability (via H^∞). Hence mixed-norm synthesis provides a technique for trading off performance and robustness, a fundamental objective in control design. The H^2/H^∞ mixed-norm problem has been addressed in a variety of settings. The treatment by Bernstein and Haddad utilized an H^2 cost bound as the basis for an auxiliary minimization problem. Necessary conditions for optimality within a full- and reduced-order fixed-structure setting were then used to characterize feedback control gains. In related work of Glover and Mustafa, the H^2 cost bound in the case of equalized H^2 and H^∞ performance weights was shown to be equal to an entropy cost functional. The centralized controller was then shown to yield a full-order controller that optimizes the entropy cost. Despite the existence of theoretical formulations, practical and efficient numerical algorithms for solving the problem were still lacking. This study developed numerical algorithms for solving the mixed-norm H^2/H^∞ problem addressed by Bernstein and Haddad.

The problem can be stated as follows: given the n-th order stabilizable and detectable plant

$$\dot{x}(t) = A\,x(t) + B\,u(t) + D_1 w(t), \qquad y(t) = C\,x(t) + D_2 w(t), \tag{1}$$

where $A \in E^{n\times n}$, $B \in E^{n\times m}$, $C \in E^{l\times n}$, $D_1 \in E^{n\times p}$, $D_2 \in E^{l\times p}$, $D_1 D_2^T = 0$, and $w(t)$ is p-dimensional white noise, find a n_c-th order dynamic compensator

$$\dot{x}_c(t) = A_c\,x_c(t) + B_c\,y(t), \qquad u(t) = C_c\,x_c(t), \tag{2}$$

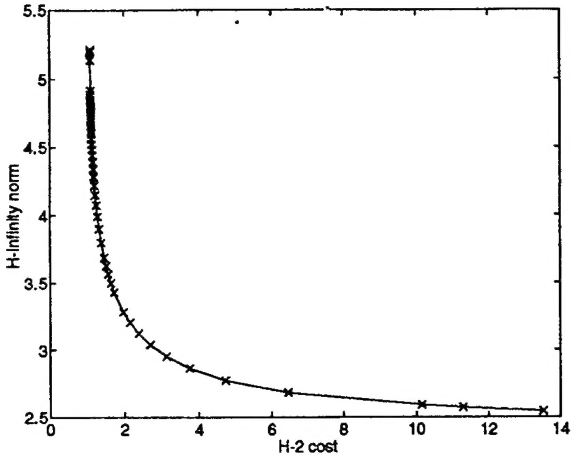

Fig. 1. Trade-off curve.

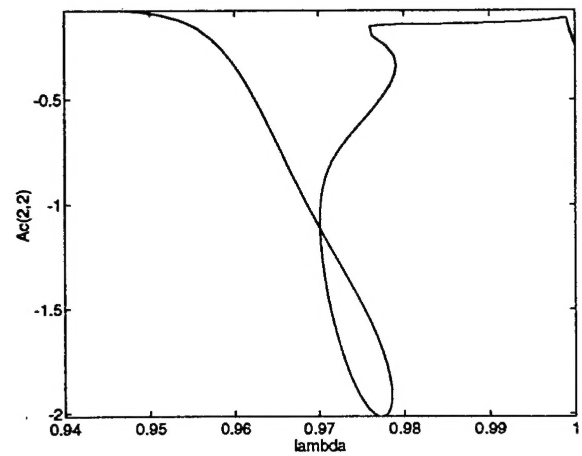

Fig. 2. Homotopy curve.

where $A_c \in E^{n_c \times n_c}$, $B_c \in E^{n_c \times l}$, $C_c \in E^{m \times n_c}$, and $n_c \leq n$, which satisfies the following criteria:

(i) the closed-loop system (1)–(2) is asymptotically stable, i.e., $\tilde{A} = \begin{pmatrix} A & BC_c \\ B_cC & A_c \end{pmatrix}$ is asymptotically stable;

(ii) the $q_\infty \times p$ closed-loop transfer function $H(s) \equiv \tilde{E}_\infty (sI_{\tilde{n}} - \tilde{A})^{-1}\tilde{D}$, from $w(t)$ to $z(t) = E_{1\infty}x(t) + E_{2\infty}u(t)$, where $\tilde{E}_\infty = (\, E_{1\infty} \quad E_{2\infty}C_c \,)$ ($E_{1\infty} \in E^{q_\infty \times n}$, $E_{2\infty} \in E^{q_\infty \times m}$, $E_{1\infty}^T E_{2\infty} = 0$), $\tilde{n} = n + n_c$, and $\tilde{D} = \begin{pmatrix} D_1 \\ B_cD_2 \end{pmatrix}$, satisfies the constraint $\|H(s)\|_\infty \leq \gamma$, where $\gamma > 0$ is a given constant; and

(iii) the performance functional $J(A_c, B_c, C_c) \equiv \lim_{t\to\infty} \mathcal{E}\left[x^T(t)R_1x(t) + u^T(t)R_2u(t)\right]$ is minimized, where $\mathcal{E}$ is the expected value, $R_1 = E_1^TE_1 \in E^{n\times n}$ and $R_2 = E_2^TE_2 \in E^{m\times m}$ ($E_1 \in E^{q\times n}$, $E_2 \in E^{q\times m}$, $E_1^TE_2 = 0$) are, respectively, symmetric positive semidefinite and symmetric positive definite weighting matrices.

Homotopy algorithms for both full- and reduced-order LQG controller design problems with an H^∞ constraint on disturbance attenuation were developed. The numerical algorithm, based on homotopy theory, solved the necessary conditions for a minimum of the upper bound on H^2 performance. The algorithms were based on two minimal parameter formulations: Ly, Bryson, and Cannon's 2×2 block parametrization and the input normal Riccati form parametrization. An over-parametrization formulation was also proposed. Numerical experiments suggested that the combination of a globally convergent homotopy method and a minimal parameter formulation applied to the upper bound minimization gave excellent results for mixed-norm H^2/H^∞ synthesis (see Fig. 1). Fig. 1 is the trade-off curve for a reduced-order LQG controller synthesis problem. The nonmonotonicity of homotopy zero curves (see Fig. 2) was demonstrated, proving that algorithms more sophisticated than standard continuation were necessary.

Modified Riccati equations.

Another prototype problem is the matrix Riccati equation,

$$AX + XA^t - X\Sigma X + R + F(X) = 0,$$

where $F(X)$ is a symmetric, low rank matrix function. Existing linear algebra techniques for the case $F(X) \equiv 0$ do not generalize well to $F(X) \not\equiv 0$, and the embedding approaches proposed so far assume $X = X(\lambda)$ rather than the more general $X = X(s)$, $\lambda = \lambda(s)$ (where λ is the homotopy parameter and s is arc length along a homotopy zero curve). This is significant, because the latter (modern homotopy) situation requires considerably more sophisticated numerical linear algebra techniques than the more restrictive situation $X = X(\lambda)$ (continuation). Using tensor products and orthogonal decompositions, some progress has been made on parallel probability-one homotopy algorithms for the Riccati equation.

Journal articles published and submitted during the grant period are:

Y. Mainguy, J. B. Birch, and L. T. Watson, "A robust variable order facet model for image data", *Machine Vision Appl.*, to appear.

C. J. Ribbens, L. T. Watson, and C. Y. Wang, "Steady viscous flow in a triangular cavity", *J. Comput. Phys.*, 121 (1994) 173–181.

Y. Ge, E. G. Collins, Jr., L. T. Watson, and L. D. Davis, "An input normal form homotopy for the L^2 optimal model order reduction problem", *IEEE Trans. Automat. Control*, 39 (1994) 1302–1305.

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Y. Ge, E. G. Collins, Jr., and L. T. Watson, "A comparison of homotopies for alternative formulations of the L^2 optimal model order reduction problem", *J. Comput. Appl. Math.*, submitted.

Y. Ge, L. T. Watson, E. G. Collins, Jr., and D. S. Bernstein, "Globally convergent homotopy algorithms for the combined H^2/H^∞ model reduction problem", *J. Math. Systems, Estimation, Control*, to appear.

N. Kogiso, L. T. Watson, Z. Gürdal, and R. T. Haftka, "Genetic algorithms with local improvement for composite laminate design", *Structural Optim.*, 7 (1994) 207–218.

Y. Chen and L. T. Watson, "Optimal trajectory planning for a space robot docking with a moving target via homotopy algorithms", *J. Robotic Sys.*, to appear.

Y. Ge, L. T. Watson, E. G. Collins, Jr., and D. S. Bernstein, "Probability-one homotopy algorithms for full and reduced order H^2/H^∞ controller synthesis", *Optimal Control Appl. Methods*, submitted.

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B. B. Lowekamp, L. T. Watson, and M. S. Cramer, "The cellular automata paradigm for the parallel solution of heat transfer problems", *Numer. Methods Heat Fluid Flow*, submitted.

S. Nagendra, D. Jestin, Z. Gürdal, R. T. Haftka, and L. T. Watson, "Improved genetic algorithm for the design of stiffened composite panels", *Comput. & Structures*, submitted.

W. I. Thacker, C. Y. Wang, and L. T. Watson, "The global stability of a rigid solid supported by elastic columns", *Internat. J. Solids and Structures*, submitted.

M. S. Cramer, B. B. Lowekamp, and L. T. Watson, "Nonlinear thermal waves: part II—analytical solutions for pulses", *Internat. J. Heat Mass Transfer*, submitted.

M. Sosonkina, L. T. Watson, and D. E. Stewart, "Note on the end game in homotopy zero curve tracking", *ACM Trans. Math. Software*, submitted.

Graduate students supported during the period of the grant are Kelly O'Brien, Yuan Wang, Susan Burgee, Andrew Southwick, William McQuain, and Yuzhen Ge, who completed her Ph.D. thesis "Homotopy algorithms for H^2/H^∞ control analysis and synthesis" in December, 1993. Ge won the Computer Science Commencement Research Award for the best thesis research in the department at the Computer Science Department Commencement in May, 1994.

Conferences attended and refereed conference proceedings during the grant period are:

- Scalable High Performance Computing Conference, Knoxville, TN, May, 1994 (2 papers).
- American Control Conference, Baltimore, MD, June, 1994.
- Nonlinear Circuit Analysis and Simulation Workshop, AT&T Bell Laboratories, Murray Hill, NJ, August, 1994.
- 15th International Symposium on Mathematical Programming, Ann Arbor, MI, August, 1994 (3 papers).
- Fifth AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Panama City, FL, Sept., 1994.
- 33rd IEEE Conference on Decision and Control, Lake Buena Vista, FL, December, 1994.
- 7th SIAM Conference on Parallel Processing for Scientific Computing, San Francisco, CA, February, 1995 (2 papers).
- NASA Computational Aerosciences Workshop, NASA Ames Research Center, Moffett Field, CA, March, 1995.
- ICASE/LaRC Workshop on Multidisciplinary Design Optimization, Hampton, VA, March, 1995.
- SIAM Southeastern-Atlantic Section Annual Meeting, Charleston, SC, March, 1995.
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S. Burgee, A. A. Giunta, R. Narducci, L. T. Watson, B. Grossman, and R. T. Haftka, "A coarse grained variable-complexity approach to MDO for HSCT design", in *Parallel Processing for Scientific Computing*, D. H. Bailey, P. E. Bjørstad, J. R. Gilbert, M. V. Mascagni, R. S. Schreiber, H. D. Simon, V. J. Torczon, and L. T. Watson (eds.), SIAM, Philadelphia, PA, 1995, 96–101.

Consulting on homotopy methods has been done with Richard Erwin, Kirtland AFB; Dr. Emmanuel Collins and Dr. Larry Davis, Harris Corporation, Melbourne, FL; Dr. Dennis Bernstein, University of Michigan, Ann Arbor; Dr. Alexander Morgan, General Motors Research Laboratories, Warren, MI; Dr. Thomas Huang, University of Illinois; Drs. Robert Melville, Mike Fang, Peter Feldman, and Ljiljana Trajkovic, AT&T Bell Laboratories, Murray Hill, NJ.

Technology transfer to industry.

Probability-one homotopy algorithms and versions of HOMPACK are in use at General Motors Research Laboratories (CAD/CAM, robotics, combustion chemistry, mechanism analysis), Harris Corporation (control systems), NASA Langley (variable geometry truss design, integrated control-structure optimization), and AT&T Bell Laboratories (circuit design, computer vision). The resultant programs from research conducted in collaboration with Emmanuel Collins and Larry Davis of Harris Corporation (control systems), Richard Erwin at Kirtland AFB, and Dennis Bernstein at the University of Michigan have been integrated into a preliminary version of a MATLAB toolbox, which has been installed at Harris Corporation, Kirtland AFB, the Interdisciplinary Center for Applied Mathematics (ICAM) at Virginia Tech, and at the University of Michigan.

Future work.

1. Homotopy convergence theory for model order reduction problems and reduced-order control problems. Rigorous proof of convergence of the homotopy algorithm is still lacking. Convergence proofs will require fundamental theory of homotopy methods and exploit the specific structure of control system problems.

2. Initial point selection. The present algorithm for choosing the initial point for the mixed norm reduced-order control problem is not satisfactory since it leads to a long homotopy path. More investigation is needed to find an overall efficient numerical algorithm.

3. Parallel Riccati equation solver. Although there are numerous algorithms for solving Riccati equations, there still remains a need for algorithms which can be generalized easily to solve variants of Riccati equations and which can operate efficiently on parallel machines. Homotopy-based algorithms may be ideal for such an application.

4. Parallel code. Practical applications often lead to large systems of equations. For these systems, parallel computers may be the only feasible computational tool. Parallel algorithms for the combined H^2/H^∞ model order reduction problem and the mixed norm control problem are needed to meet the computational demand of realistic applications.

5. MATLAB package. Considerably more time and effort are needed to make the MATLAB toolbox resulting from this work more portable and robust. New capabilities will also be added to this toolbox.